

Name: Solutions

Collaborator(s): _____

Please take your time and answer each question clearly and carefully. You may work with other students, but please be sure to write your own version of your solutions, in your own words, on this sheet. Please note your collaborators above. Collaboration is optional, but please spend your time constructively.

Consider the following two functions:

$$f(t) = 20 + 2t - t^2$$

$$g(t) = 4t + 3$$

1. Compute
- f
- and
- g
- for integer values 0 through 5.

$$\begin{aligned} f(0) &= 20 + 2 \cdot 0 - 0^2 = 20 & f(5) &= 20 + 2 \cdot 5 - 5^2 = 5 \\ f(1) &= 20 + 2 \cdot 1 - 1^2 = 21 & g(0) &= 4 \cdot 0 + 3 = 3 \\ f(2) &= 20 + 2 \cdot 2 - 2^2 = 20 & g(1) &= 4 \cdot 1 + 3 = 7 \\ f(3) &= 20 + 2 \cdot 3 - 3^2 = 17 & g(2) &= 4 \cdot 2 + 3 = 11 \\ f(4) &= 20 + 2 \cdot 4 - 4^2 = 12 & g(3) &= 4 \cdot 3 + 3 = 15 \end{aligned}$$

$$\begin{aligned} g(4) &= 19 \\ g(5) &= 23 \end{aligned}$$

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2. Find the average rate of change for both
- f
- and
- g
- over the following intervals, using your data above:

(a) $[0, 2]$

$$\frac{f(2) - f(0)}{2 - 0} = \frac{20 - 20}{2 - 0} = \frac{0}{2} = 0$$

(b) $[0, 4]$

$$\frac{f(4) - f(0)}{4 - 0} = \frac{12 - 20}{4 - 0} = \frac{-8}{4} = -2$$

(c) $[1, 5]$

$$\frac{f(5) - f(1)}{5 - 1} = \frac{5 - 21}{5 - 1} = \frac{-16}{4} = -4$$

(d) $[2, 3]$

$$\frac{f(3) - f(2)}{3 - 2} = \frac{17 - 20}{3 - 2} = \frac{-3}{1} = -3$$

$$\frac{g(2) - g(0)}{2 - 0} = \frac{11 - 3}{2 - 0} = \frac{8}{2} = 4$$

$$\frac{g(4) - g(0)}{4 - 0} = \frac{19 - 3}{4 - 0} = \frac{16}{4} = 4$$

$$\frac{g(5) - g(1)}{5 - 1} = \frac{23 - 7}{5 - 1} = \frac{16}{4} = 4$$

$$\frac{g(3) - g(2)}{3 - 2} = \frac{15 - 11}{3 - 2} = \frac{4}{1} = 4$$

TURN OVER

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3. What observation can you make about $g(t)$ based on your answers to the previous question?

The rate is the same because $g(t)$ is linear.

4. For $f(t)$ only, estimate the instantaneous slope at the indicated points, using the two-slope-average formula (below).

$$\frac{1}{2} \left(\frac{y_3 - y_2}{t_3 - t_2} + \frac{y_2 - y_1}{t_2 - t_1} \right)$$

(a) $f'(1)$

$$\frac{1}{2} \left(\frac{f(2) - f(1)}{2 - 1} + \frac{f(1) - f(0)}{1 - 0} \right) = \frac{1}{2} \left(\frac{20 - 21}{2 - 1} + \frac{21 - 20}{2 - 1} \right)$$

$$= \frac{1}{2} \left(\frac{-1}{1} + \frac{1}{1} \right) = \frac{1}{2} (-1 + 1) = 0$$

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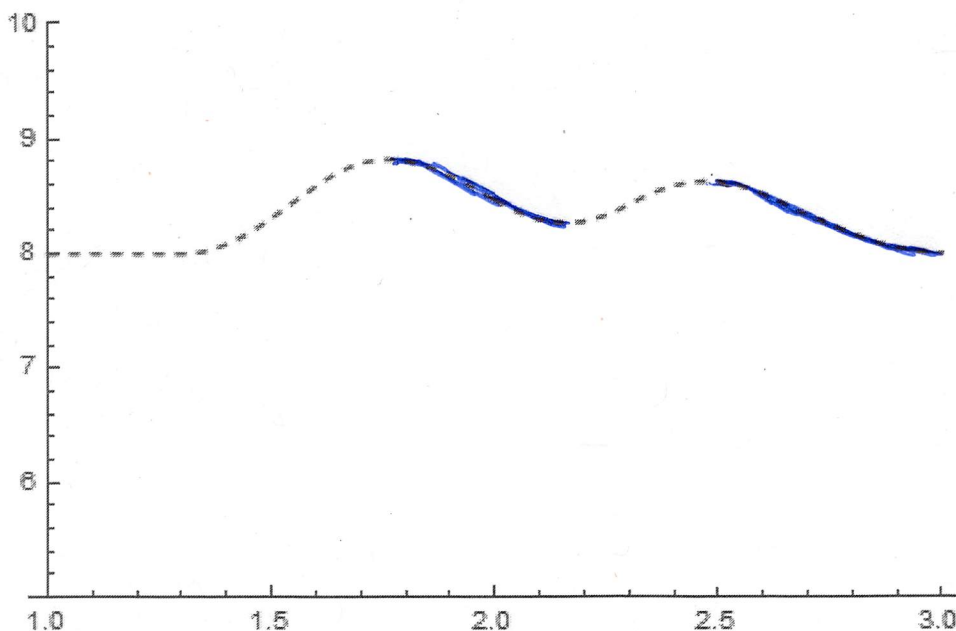
(b) $f'(3)$

$$\frac{1}{2} \left(\frac{f(4) - f(3)}{4 - 3} + \frac{f(3) - f(2)}{3 - 2} \right) = \frac{1}{2} \left(\frac{12 - 17}{4 - 3} + \frac{17 - 20}{3 - 2} \right)$$

$$= \frac{1}{2} \left(\frac{-5}{1} + \frac{-3}{1} \right) = \frac{1}{2} (-5 - 3) = -4$$

↑ ↑
-5 and -3
are two slopes
we are averaging.

5. Consider my fictional cat and his fluctuating weight. Shade in the parts of this graph where the rate of change of the cat's weight is negative.



Recall:
Negative rate of
change
means decreasing.