MTH 152 In-Class 3 – September 7 Fall 2017

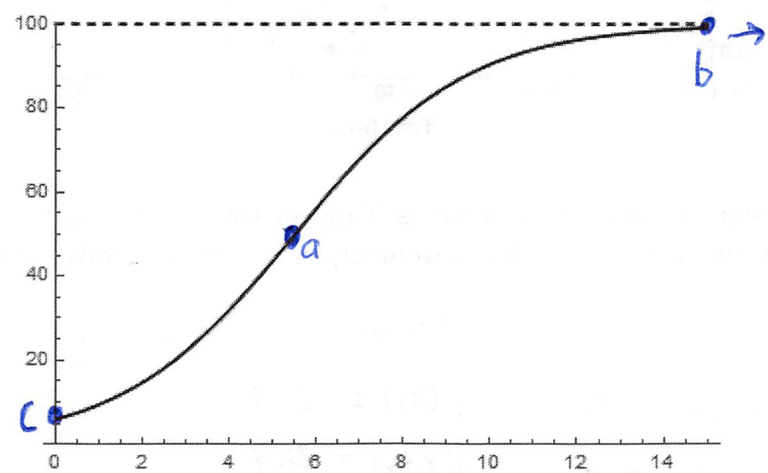
Name: Solutions

Collaborator(s):

Please take your time and answer each question clearly and carefully. You may work with other students, but please be sure to write your own version of your solutions, in your own words, on this sheet. *<u>Please note your collaborators above.</u>* *Collaboration is optional, but please spend your time constructively.*

1. Consider our old population model, which had solutions that may look like this:

The methods of calculus can be used to determine these precisely!



Based on this particular example graph, answer the following questions. Use sentences, and indicate the relevant locations on the graph. Explain how you know your answers.

(a) When (what t value) is the rate of change $P'(t)$ largest?

(b) When (what t value) is the rate of change $P'(t)$ smallest?

(c) When is the relative rate of change $P'(t)/P(t)$ largest?

(d) How do your answers to (a) and (c) compare?

(a) The rate of change is greatest where the graph is most steep, which occurs at approximately $t = 5.5$, and more insightfully, at $P(t) = K/2$, half the carrying capacity.

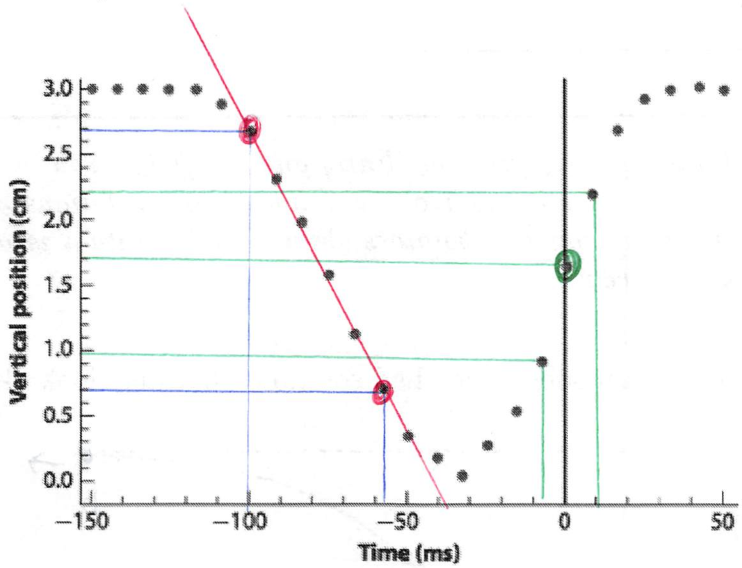
(b) The rate of change will decrease as $t \to \infty$, so here we pick the rightmost value of t — if we could go further, we would.

(c) This actally occurs right at $t = 0$. While it's not easy to see on a graph, conceptually the populaton grows more if there is more unused "capacity" in the environment.

(d) They are very different!

TURN OVER

2. A veterinary scientist measures the position of a cat's tongue as it drinks water. The data are shown in the plot below.



Note: You could use different points and different types of estimates. Answers will vary.

Estimate the average speed of the tongue as it moves towards the water (downward), and then repeat your estimate for the movement away from the water (upward).

Downward: Using a line through two points (red).

$t_1 = -100 \qquad f(t_1) = 2.7$

$t_2 = -58 \qquad f(t_2) = 0.7$

$$f'(t) \approx \frac{f(t_2)-f(t_1)}{t_2 - t_1} = \frac{0.7-2.7}{-58-(-100)} = \frac{-2}{42} \approx \boxed{0.0476 \text{ cm/ms}}$$

Upward: Using one point ($t=0$) and averaging two slopes (green).

$t_1 = -8 \qquad f(t_1) = 1$

$t_2 = 0 \qquad f(t_2) = 1.7$

$t_3 = 10 \qquad f(t_3) = 2.2$

$$\frac{f(t_2)-f(t_1)}{t_2-t_1} = \frac{1.7-1}{0-(-8)} = \frac{0.7}{8} = 0.0875$$

$$\frac{f(t_3)-f(t_2)}{t_3-t_2} = \frac{2.2-1.7}{10-0} = \frac{0.5}{10} = 0.05$$

Average: $\frac{1}{2}(\text{slope 1} + \text{slope 2}) = \frac{1}{2}(0.0875+0.05) = \boxed{0.06875 \frac{\text{cm}}{\text{ms}}}$

Based on an exercise by Dr. Lou Gross, itself based on Reis et al. "How Cats Lap: Water ~~Update~~ Uptake by Felis Catus." Science 330:1231-1234.