Name: Solutions

Collaborator(s):

Please take your time and answer each question clearly and carefully. You may work with other students, but please be sure to write your own version of your solutions, in your own words, on this sheet. Please note your collaborators above. Collaboration is optional, but please spend your time constructively.

1. Consider a population model where the size of a town's population is given by:

$$P(t) = 16250 \cdot (0.87)^t,$$

where t is measured in years since 1985.

(a) What is the rate of change of the population, as a function of t?

Recall $a^{kt} \mapsto k \ln(a)\, a^{kt}$

$P'(t) = 16250 \cdot \left(\ln(0.87) \cdot (0.87)^t \right)$

$\approx -2263 \cdot (0.87)^t$ people/year

(b) What is the rate of change of P in the year 2000? End with a numerical answer. (Use a calculator of some kind. Ask a neighbor or me if you don't have one).

$P'(15) \approx -2263 \cdot (0.87)^{15} \approx -280.2$ people/year

Note $t = 15$ is 2000, 15 years after 1985.

(c) Same question, for 2010? How does this compare to (b)?

$P'(25) \approx -2263 \cdot (0.87)^{25} \approx -69.6$ ppl/yr

(Compare: $P(15) \approx 2012$ people
$P(25) \approx 500$ people)

(d) Based on your answers above, what is the long-term behavior of $P(t)$? What is another way you could determine this (we learned it a few chapters ago)?

The population is always decreasing ($P'(t) < 0$). It might go to zero? (In fact, it does. This is confirmed by $\lim_{t\to\infty} P(t) = 0$.)

TURN OVER

2. Using the rules we have just learned, give the derivatives of the following functions:

(a) $f(t) = 3t^2 + 2t - te^t$ — Product Rule

Building blocks:

$t^2 \mapsto 2t$

$t \mapsto 1$

$e^t \mapsto e^t$

$$f'(t) = 3(2t) + 2(1) - (t \cdot e^t + 1 \cdot e^t)$$
$$= 6t + 2 - te^t - e^t$$

(b) $g(x) = 3k\sin(x) + 3$
Assume k is a constant.

$\sin(x) \mapsto \cos(x)$

$3 \mapsto 0$

$$g'(x) = 3k\cos(x) + 0$$
$$= 3k\cos(x)$$

(c) $y = e^x\cos(x) + \cos(x)$

product rule

$\cos(x) \mapsto -\sin(x)$

$$y' = e^x(-\sin(x)) + e^x\cos(x) + (-\sin(x))$$
$$= -e^x\sin(x) + e^x\cos(x) - \sin(x)$$

(d) If $P(t) = 600e^t$, find $\frac{P'(t)}{P(t)}$.
Remember that we called this the relative growth rate.

$e^t \mapsto e^t$, so $P'(t) = 600e^t$, so $\frac{P'(t)}{P(t)} = 1.$

Note: If t is time in years, P a population of people,
Units are: $P'(t)$ $\frac{\text{people}}{\text{year}}$

$$\frac{P'(t)}{P(t)} \quad \frac{\frac{\text{people}}{\text{year}}}{\text{people}} = \frac{1}{\text{year}}$$

Problem 1 based on an exercise by Dr. Lou Gross.