Name: Solutions

Collaborator(s):

Please take your time and answer each question clearly and carefully. You may work with other students, but please be sure to write your own version of your solutions, in your own words, on this sheet. Please note your collaborators above. Collaboration is optional, but please spend your time constructively.

1. Newton's Law of Cooling helps us determine the time of death when a human body is discovered. It states that at a fixed ambient temperature T_∞, an object with temperature $T(t)$ has rate of change $T'(t) = k(T(t) - T_\infty)$.

   This allows us to determine that an object cools from its initial value $T_0 = T(0)$ to the room temperature $T_\infty = \lim_{t\to\infty} T(t)$ as exponential decay:

   $$T(t) = (T_0 - T_\infty)e^{-kt} + T_\infty.$$

   My coffee was 100° F when class began. One hour later, it is now 80° F. The classroom is 70° F. When did I make the coffee, if the coffeemaker produces coffee at 150° F?

   Class started at 9:40 – it may be easier to consider this time $t = 0$. Also, recall: to solve $e^x = y$ for x, you take the natural log (ln)of both sides and obtain $x = \ln(y)$.

$T_0 = 100, \quad T_\infty = 70.$

$T(t) = 30e^{-kt} + 70$

Here it's good to double-check $T(0) = 100$ and $\lim_{t\to\infty} T(t) = 70$.

$T(1) = 30e^{-k\cdot 1} + 70 = 80$ (solving for k.)

$30e^{-k} = 10$

$e^{-k} = \frac{1}{3}$

$e^{k} = 3$

$k = \ln(3)$

Solving for t_{cm}, time of coffee making.

$T(t_{cm}) = 30e^{-\ln(3)t_{cm}} + 70 = 150$

$30e^{-\ln(3)t_{cm}} = 80$

$e^{-\ln(3)t_{cm}} = \frac{8}{3}$

$e^{\ln(3)t_{cm}} = \frac{3}{8}$

$\ln(3)t_{cm} = \ln(3/8)$

$t_{cm} = \frac{\ln(3/8)}{\ln(3)}$

≈ -0.893 hrs.

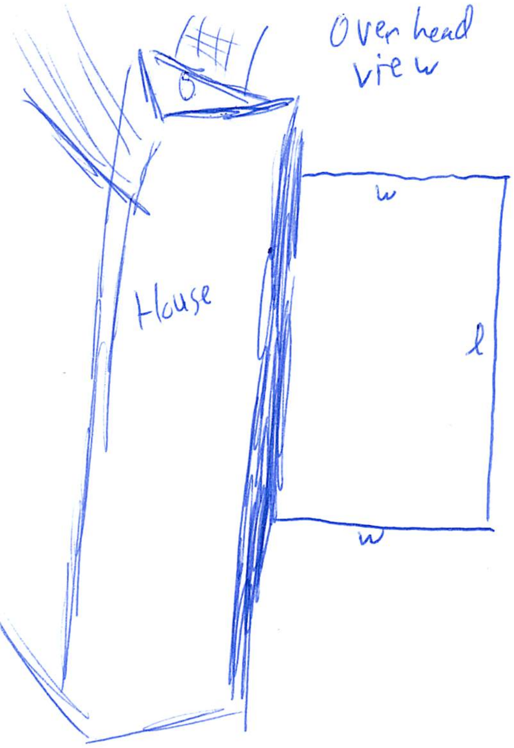
$\boxed{\text{53 min before class, 8:47, approximately}}$

TURN OVER

2. You are building a fence that backs up onto your house. The length of the fence will run along the house (you don't build fence on that side), and its width will extend away from your house, forming a rectangle – three sides fence, one side house.

   If you have 500ft of fence, what dimensions of the fence will maximize the area enclosed by the fence.

   Note: This is not the same as building a fence in the open. Your answer will be different from the example we did in class.



$A = \ell \cdot w$

$500 = \ell + 2w$ (not $2\ell + 2w$)

$\ell = 500 - 2w$ — you could eliminate ℓ, or w, either works.

$A = (500-2w)\cdot w$

$A(w) = 500w - 2w^2$ — Now it's a function of w.

$A'(w) = 500 - 4w = 0$ — critical point(s)

$500 = 4w$

$125 = w \rightarrow \ell = 500 - 2w = 250$

$\boxed{\ell = 250, \quad w = 125}$

Check

$A'(100) = 100$ $\quad$ $A'(150) = -100$

$+$ $\quad$ MAX $\quad$ $-$ $\quad$ A'

$w = 125$

(check isn't needed if you notice $A(w)$ is just a parabola!)

If you had 1000ft of fence instead, how would your answer change? Offer a reason that does not require repeating your solution above with a different value.

It's twice as large – but the ratios stay the same. $(\ell = 500, \ w = 250)$