

Name: Solutions

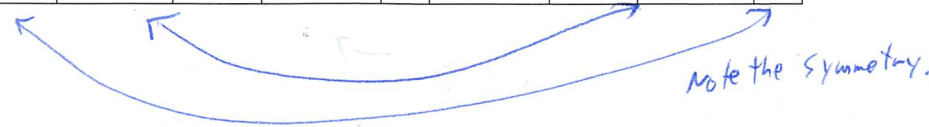
Collaborator(s): _____

Please take your time and answer each question clearly and carefully. You may work with other students, but please be sure to write your own version of your solutions, in your own words, on this sheet. Please note your collaborators above. Collaboration is optional, but please spend your time constructively.

For the length of this activity, consider our example function $f(x) = 1 - x^2$.

1. Fill in the missing values of the table. *(Just plug in.)*

| | | | | | | | | | |
|------|----|--------|------|--------|---|--------|------|--------|---|
| x | -1 | -0.75 | -0.5 | -0.25 | 0 | 0.25 | 0.5 | 0.75 | 1 |
| f(x) | 0 | 0.4375 | 0.75 | 0.9375 | 1 | 0.9375 | 0.75 | 0.4375 | 0 |



2. Use the above data to approximate the area under the curve, from $x = -1$ to $x = 1$, using the left and right endpoint rules, using $n = 4$ and $n = 8$. (That means you will have four answers here.)

n=4 $\Delta x = 0.5$

$f(x_0) = 0$
 $f(x_1) = 0.75$
 $f(x_2) = 1$
 $f(x_3) = 0.75$
 $f(x_4) = 0$

→ Left

→ Right

n=8 $\Delta x = 0.25$

$f(x_0) = 0$
 $f(x_1) = 0.4375$
 $f(x_2) = 0.75$
 $f(x_3) = 0.9375$
 $f(x_4) = 1$
 $f(x_5) = 0.9375$
 $f(x_6) = 0.75$
 $f(x_7) = 0.4375$
 $f(x_8) = 0$

→ Left

→ Right

Add up each list, multiply by Δx . You get

- $n = 4$, Left: 1.25
- $n = 4$, Right: 1.25
- $n = 8$, Left: 1.3125
- $n = 8$, Right: 1.3125

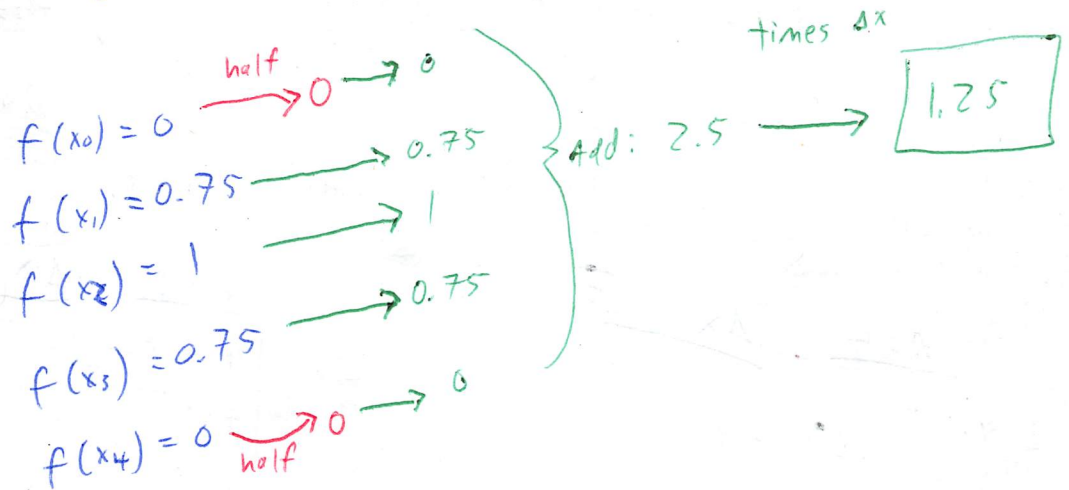
Each pair is the same because $f(x_0) = f(x_4)$ in $n=4$ or $f(x_8)$ in $n=8$ (Due to symmetry.)

TURN OVER

3. Repeat this approximation for $n = 4$ using the trapezoidal rule.

| | Average | Times $\Delta x = 0.5$ | |
|-----------------|---------|------------------------|---|
| $f(x_0) = 0$ | 0.375 | 0.1875 | } Total: 1.25 |
| $f(x_1) = 0.75$ | 0.875 | 0.4375 | |
| $f(x_2) = 1$ | 0.875 | 0.4375 | |
| $f(x_3) = 0.75$ | 0.375 | 0.1875 | |
| $f(x_4) = 0$ | | | |

Note: We saw this can be done ~~fast~~er:



We also found that $\frac{\text{Left} + \text{Right}}{2} = \text{Trapezoid}$,

so we could have just averaged our answers from #1.