MTH 152 In-Class 7 – October 19 Fall 2017

Name: Solutions

Collaborator(s): ______

Please take your time and answer each question clearly and carefully. You may work with other students, but please be sure to write your own version of your solutions, in your own words, on this sheet. Please note your collaborators above. Collaboration is optional, but please spend your time constructively.

1. Find the anti-derivative (indefinite integral) of each function. Include $+C$ for each.

   (a) If $h(t) = 2t + 1$, find $H(t)$ so that $H'(t) = h(t)$:

   Power Rule: $2t^1 + 1t^0 \mapsto 2(\frac{1}{2}t^2) + 1(\frac{1}{1}t^1)$

   $\boxed{H(t) = t^2 + t + C}$

   (b) $\int \left(\frac{3}{z} + z\right) dz$ $\longrightarrow$ $\boxed{3\ln(|z|) + \frac{1}{2}z^2 + C}$

   Power rule $z \rightarrow \frac{1}{2}z^2$

   BUT $z^{-1} \rightarrow \cancel{\frac{1}{0}z^0}$? NO

   ($\frac{1}{z}$ is instead $\ln(z)$)

   (c) If $g(x) = \cos(3x)$, find $G(x)$ so that $G'(x) = g(x)$:

   Deriv: $\sin(x) \rightarrow \cos(x)$

   So we reverse it. But what about "3"?

   divide by 3

   Deriv: $\sin(3x) \rightarrow 3\cos(3x)$

   So too: $\boxed{\frac{1}{3}\sin(3x)} \rightarrow \cos(3x)$

   $\boxed{G(x) = \frac{1}{3}\sin(3x) + C}$

   (d) $\int 10e^{-2t}dt$

   Similar to above:

   Deriv: $e^{-2t} \rightarrow -2e^{-2t}$

   multiply by -5:

   $\boxed{-5e^{-2t}} \rightarrow 10e^{-2t}$

   $\boxed{-5e^{-2t} + C}$

TURN OVER

2. Find each indicated area (definite integral). Use your answers from the previous problem. (Recall that the $+C$ does not affect your answer here.)

(a) $\int_1^3 (2t+1)dt$

$H(t) = t^2 + t$

$H(3) = 3^2 + 3 = 12$

$H(1) = 1^2 + 1 = 2$

$H(3) - H(1) = \boxed{10}$

(b) The area under $f(z) = \frac{3}{z} + z$ from $z = 1$ to $z = 3$.

$F(z) = 3\ln(|z|) + \frac{1}{2}z^2$

$F(3) = 3\ln(3) + \frac{1}{2}\cdot 3^2$

$= 3\ln(3) + \frac{9}{2}$

$F(1) = 3\ln(1) + \frac{1}{2}\cdot 1^2$

$= 0 + \frac{1}{2}$

$\boxed{F(3) - F(1) = 3\ln(3) + 4}$

(c) $\int_{\pi/6}^{\pi/2} \cos(3x)dx$

$G(x) = \frac{1}{3}\sin(3x)$

$G(\pi/2) = \frac{1}{3}\sin(3\pi/2) = -1/3$

$G(\pi/6) = \frac{1}{3}\sin(3\pi/6) = 1/3$

$G(\pi/2) - G(\pi/6) = -1/3 - 1/3 = \boxed{-2/3}$

(d) $\int_0^5 10e^{-2t}dt$

$F(t) = -5e^{-2t}$

$F(5) = -5e^{-2\cdot 5} = -5e^{-10}$

$F(0) = -5e^{-2\cdot 0} = -5e^0 = -5$

$F(5) - F(0) = -5e^{-10} - (-5) = \boxed{5 - 5e^{-10}}$