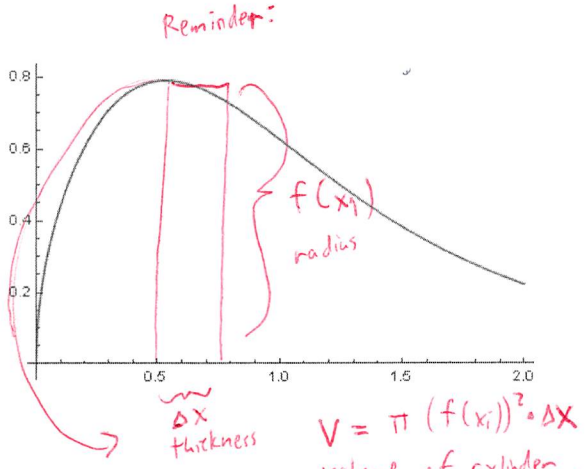
MTH 152 | In-Class 9 – November 2 | Fall 2017

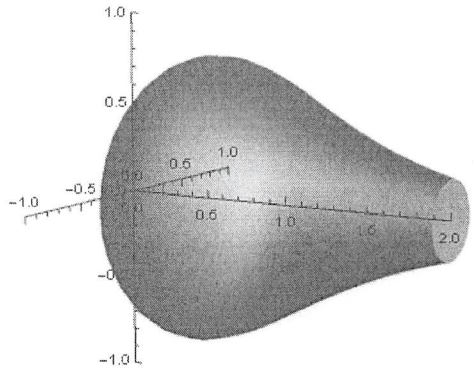
Name: Solutions

Collaborator(s): _____

Please take your time and answer each question clearly and carefully. You may work with other students, but please be sure to write your own version of your solutions, in your own words, on this sheet. Please note your collaborators above. Collaboration is optional, but please spend your time constructively.

1. The function $f(t) = \frac{10}{\sqrt{\pi}}\sqrt{t}\ (t^2+2)^{-2}$ is rotated around the t-axis, producing a solid with the following shape, considering $0 \le t \le 2$. The images below show $f(t)$ vs. t (left) as well as the rotated solid figure (right).





(a) Set up the integral to find the volume of this solid. Use the formula $\int \pi(f(t))^2 dt$.

Note: Here you should just set up and simplify the problem. Don't integrate.

$$\int_0^2 \pi \left(\frac{10}{\sqrt{\pi}}\sqrt{t}\left(t^2+2\right)^{-2}\right)^2 dt$$

$$= \int_0^2 \pi \ \frac{100}{\pi} \cdot t \cdot \left(t^2+2\right)^{-4} dt$$

$$= \int_0^2 100\, t\, \left(t^2+2\right)^{-4} dt$$

TURN OVER

(b) Now that you've set up the integral in part (a), find the volume by integrating.

You should be able to do this integral. The simplified answer you got in part (a) should really look like something you can handle. If it doesn't, check your work in part (a) and/or ask for help.

$\int_0^2 100t(t^2+2)^{-4}\,dt$

$\hookrightarrow u = t^2+2$

$du = 2t\,dt$ (looks good)

$x=0 \to u=2$

$x=2 \to u=6$

$= \int_2^6 50u^{-4}\,du$

$= \left[50\left(\frac{1}{-3}u^{-3}\right)\right]_{u=2}^{6}$

we don't need +C for definite integrals (areas). If we used +C, it would cancel.

This reminds us the next step is F(b)−F(a).

$= \left[-\frac{50}{3}u^{-3}\right]_{u=2}^{6}$

$= -\frac{50}{3}\cdot 6^{-3} - \left(-\frac{50}{3}\cdot 2^{-3}\right) = \frac{50}{3\cdot 2^3} - \frac{50}{3\cdot 6^3} = \frac{325}{162}$

This is okay to stop.

≈ 2.00617 approx

2. Consider a cube-shaped, totally enclosed water tank, 10 inches on each side. The density of a toxic gas dissolved in the water is dependent on the depth of water. At depth d (where $0 \le d \le 10$ of course), the density of the toxic gas is $0.05\sqrt{d}$ ppm, where the unit ppm is "parts per million" (meaning 1ppm is one molecule of the gas per million molecules of water).

However, the density of the water is also dependent on the depth. At depth d, there are $10d^{3/2}$ million molecules of water.

Set up the integral that gives the total number of molecules of toxin in the tank. (You do not have to solve it, although you would be able to do so.)

$\int_0^{10} (0.05\cdot\sqrt{d})(10\cdot d^{3/2})\,dd = \boxed{\int_0^{10} 0.5d^2\,dd}$

depth $0 \le d \le 10$

parts per million

number of millions

Yes, if d is the variable, you write "dd".