MTH 152 **In-Class 10 – November 14** Fall 2017

Name: Solutions

Collaborator(s): _____

Please take your time and answer each question clearly and carefully. You may work with other students, but please be sure to write your own version of your solutions, in your own words, on this sheet. Please note your collaborators above. Collaboration is optional, but please spend your time constructively.

1. Consider a random variable X distributed according to the P.D.F. $f(x) = \frac{2}{\pi}\frac{1}{(1+x^2)^2}$.

   Find the expected value (average) of X.

$$\int_{-\infty}^{\infty} x\, f(x)\, dx = \int_{-\infty}^{\infty} x \cdot \frac{2}{\pi}\frac{1}{(1+x^2)^2}\, dx$$

Find anti-derivative:

$$\int x \cdot \frac{2}{\pi}\frac{1}{(1+x^2)^2}\, dx$$

$u = 1+x^2$

$du = 2x\, dx$

$\frac{1}{2}du = x\, dx$

$$\int \frac{2}{\pi}\frac{1}{u^2}\frac{1}{2}\, du$$

$$\frac{1}{\pi}\int u^{-2}\, du$$

$$\frac{1}{\pi}\left(\frac{1}{-1}u^{-1}\right)$$

$$-\frac{1}{\pi u}$$

$$-\frac{1}{\pi(1+x^2)}$$

Take limits:

$$\lim_{b\to\infty}\lim_{a\to-\infty}\left(-\frac{1}{\pi(1+b^2)} - \left(-\frac{1}{\pi(1+a^2)}\right)\right)$$

"$\frac{1}{\infty}$" for both

$0 - 0 = 0$

(Note: This P.D.F. is symmetrical. It makes perfect sense to have mean 0.)

TURN OVER

2. An individual is chosen at random from the US population. Which of these random variables about that person would be modeled as a continuous variable (as opposed to a discrete variable)? Your answer will be more than one choice – circle all.

- height
- number of cats owned
- average age of cats owned
- weight
- current body temperature
- shoe size
- length of foot
- number of cell phones in household
- current location (city, state)
- current location (longitude, latitude)
- number of freckles
- number of calories consumed today

3. The table below shows the standard normal distribution, $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-x^2/2} dx$ (with $\mu = 0$, $\sigma = 1$) for various values of z. Use this to compute the probability that a normally distributed random variable is within 0.5 standard deviations of its mean.

z	integral	z	integral	z	integral
−2.0	0.0227501	−0.6	0.274253	0.8	0.788145
−1.9	0.0287166	−0.5	0.308538	0.9	0.81594
−1.8	0.0359303	−0.4	0.344578	1.0	0.841345
−1.7	0.0445655	−0.3	0.382089	1.1	0.864334
−1.6	0.0547993	−0.2	0.42074	1.2	0.88493
−1.5	0.0668072	−0.1	0.460172	1.3	0.9032
−1.4	0.0807567	0.0	0.5	1.4	0.919243
−1.3	0.0968005	0.1	0.539828	1.5	0.933193
−1.2	0.11507	0.2	0.57926	1.6	0.945201
−1.1	0.135666	0.3	0.617911	1.7	0.955435
−1.0	0.158655	0.4	0.655422	1.8	0.96407
−0.9	0.18406	0.5	0.691462	1.9	0.971283
−0.8	0.211855	0.6	0.725747	2.0	0.97725
−0.7	0.241964	0.7	0.758036		

this is "F(a)"

this is "F(b)"

mean

range we want

Want: $\int_{-0.5}^{0.5} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = \int_{-\infty}^{0.5} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx - \int_{-\infty}^{-0.5} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$

$= 0.691462 - 0.308538$

$= \boxed{0.382924}$

Note: Due to a typo, data in the table was skewed. Your answer might be 0.5205 instead of this value.