MTH 152 In-Class 11 – November 21 Fall 2017

Name: Solutions

Collaborator(s):

Please take your time and answer each question clearly and carefully. You may work with other students, but please be sure to write your own version of your solutions, in your own words, on this sheet. Please note your collaborators above. Collaboration is optional, but please spend your time constructively.

1. Consider the following differential equation describing logistic growth for a population:

$$P' = rP\left(1 - \frac{P}{K}\right).$$

Determine $P(t)$ by solving the differential equation. (Use separation of variables.)

Hint: $\frac{1}{P(1-P/K)} = \frac{1}{P} - \frac{1}{P-K}$. *Also, the back has a bit of extra space if you need it.*

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

$$\frac{1}{P(1-P/K)}\,dP = r\,dt$$

$$\int \frac{1}{P(1-P/K)}\,dP = \int r\,dt$$

Hint

$$\int \left(\frac{1}{P} - \frac{1}{P-K}\right) dP = \int r\,dt$$

$$\ln(P) - \ln(P-K) = rt + C$$

back

TURN OVER

$$\ln(P) - \ln(P-K) = rt + C$$

Exponent Rules

$$e^{\ln(P) - \ln(P-K)} = e^{rt+C}$$

$$\frac{e^{\ln(P)}}{e^{\ln(P-K)}} = e^{rt} e^{C}$$

$C_2 = e^C$

$$\frac{P}{P-K} = C_2 e^{rt}$$

$$P = C_2 e^{rt}(P-K)$$

$$P = C_2 e^{rt} P - C_2 K e^{rt}$$

$$C_2 e^{rt} P - P = C_2 K e^{rt}$$

$$(C_2 e^{rt} - 1) P = C_2 K e^{rt}$$

$$P = \frac{C_2 K e^{rt}}{C_2 e^{rt} - 1}$$

$$P = \frac{K e^{rt}}{e^{rt} - 1/C_2}$$

$$\boxed{P = \frac{K e^{rt}}{e^{rt} + C_3}}$$

$C_3 = -1/C_2$

2. Given your answer $P(t)$, a function of t, from the previous question: What are the limits (horizontal asymptotes) of the function P? What does this tell you about the population in question?

$$\lim_{t \to -\infty} \frac{K e^{rt}}{e^{rt} + C_3} = \frac{K \cdot 0}{0 + C_3} = \boxed{0}$$

(unless $C_3 = 0$)

$$\lim_{t \to \infty} \frac{K e^{rt}}{e^{rt} + C_3} = \lim_{t \to \infty} \frac{K}{1 + C_3/e^{rt}} = \frac{K}{1+0} = \boxed{K}$$

The population grows up to a stable value K, and back in time, the population may not have existed.