MTH 152 In-Class 12 – November 30 Fall 2017

Name: Solutions

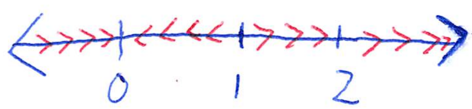
Collaborator(s):

Please take your time and answer each question clearly and carefully. You may work with other students, but please be sure to write your own version of your solutions, in your own words, on this sheet. Please note your collaborators above. Collaboration is optional, but please spend your time constructively.

For each differential equation given, find the equilibrium values of y. Then draw the "sign chart" indicating for which values of y a solution would be increasing or decreasing. Use this to classify each equilibrium point is stable, unstable, or semistable. For each stable or semistable equilibrium, which initial values $y(0)$ would tend towards that equilibrium?

1. $y' = y(y-1)(y-2)^2$

$y = 0, 1, 2$



y	y'	
-1	$-1(-2)(-3)^2 = 18$	+
$1/2$	$\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})^2 = -9/16$	−
$3/2$	$\frac{3}{2}(\frac{1}{2})(-\frac{1}{2})^2 = 3/16$	+
3	$3(2)(1)^2 = 6$	+

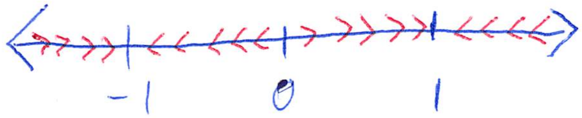
$y=0$ stable (all $y(0) < 1$)

$y=1$ unstable

$y=2$ semistable (all $1 < y(0) \le 2$)

2. $y' = y(1-y^2)$

$y = 0, 1, -1$



y	y'	
-2	$-2(1-4) = 6$	+
$-1/2$	$-\frac{1}{2}(1-\frac{1}{4}) = -3/8$	−
$1/2$	$\frac{1}{2}(1-\frac{1}{4}) = 3/8$	+
2	$2(1-4) = -6$	−

$y=-1$ stable (all $y(0) < 0$)

$y=0$ unstable

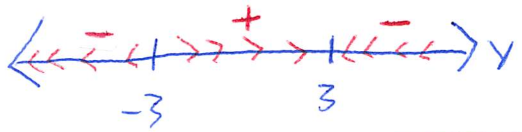
$y=1$ stable (all $y(0) > 0$)

TURN OVER

3. $y' = 9 - y^2$

$Y = 3, -3$

Y	Y'	
-5	9-25=-16	-
0	9-0=9	+
5	9-25=-16	-



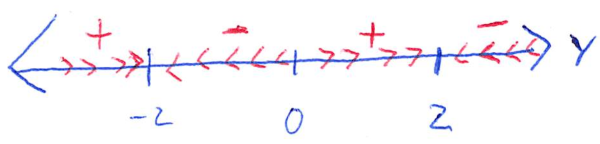
$Y = -3$ unstable

$Y = 3$ stable (all $Y(0) > 3$)

4. $y' = 4y - y^3 = Y(4 - Y^2)$

$Y = 0, 2, -2$

Y	Y'	
-3	-3(4-9) = 15	+
-1	-1(4-1) = -3	-
1	1(4-1) = 3	+
3	3(4-9) = -15	-



$Y = -2$ stable (all $Y(0) < 0$)

$Y = 0$ unstable

$Y = 2$ stable (all $Y(0) > 0$)

This is very similar to # 2. You should notice that!