This document contains general instructions for completing this assignment, followed by some background leading up to the project, and then the prompt(s) for the project. The grading rubric is also included.

First, let us remember the purposes of the projects in this course: To foster the use of computer-assisted methods in solving mathematical and scientific problems; To engage in constructive, in-depth problem solving to explore course material; and to practice writing in a technical context.

Your paper should be formatted in a reasonable way. You should include your name, the name of the class, the semester, the assignment, and a title you choose for your paper. ("Math Paper" or "Joe Student" or "Paper 2" is not a title.)

You may type your paper using any software with which you are familiar. Examples of software you could use are MS Word + Equation Editor, LaTeX, MathType, or even Mathematica or Maple (both of which have word processing features). In any event, though, your paper should be typed. You will submit a printed copy of this and any other physical supporting materials.

You will also send a PDF version of this paper, the source document (DOC, DOCX, etc.), and any other electronic materials (e.g. code, image files, etc.) by email. These should be submitted the same day as the hard-copy, the due date. The PDF is important – if you only send the DOCX file, it may not display correctly. Please send both.

Your margins, spacing, font choice, etc. should be standard: Margins should be 1" give or take, font should be a standard font in 12 point, and lines should be 1.5-, or double-spaced. Figures and equations should be included appropriately when needed. If you cannot type all your equations, you can do it the old-fashioned way and leave a blank space, then write them in pen on your print-out.

Your paper should address directly, and in depth, the question in the prompt. You should explore the question/topic in detail. The creativity, thoroughness, and correctness of your response will comprise a major component of your grade. You are required not only to find "the answer," but to discuss the solution, how you found it, and what it says about the problem (which will vary by prompt). Although you are not required to include every possible type of topic we have covered, failing to include a relevant type of method, idea, concept, visual, etc. (especially one explicitly mentioned in the prompt) may result in a deduction if your answer seems incomplete.

For computer-based portions of the work, the instructions below and everything else we've covered so far in class should be enough to get you started. If there are any questions about this, please send me an email.

Your paper should be roughly 2-3 pages in length, or about 700 words. Your paper may be longer, especially if you double-space or use a large number of images. The absolute minimum is 500 words, which should span about 1.5-2 pages (assuming one graphic). Papers below this threshold may be subject to automatic deductions. Your paper should include the appropriate amount (and balance) of mathematics and exposition.

If you are concerned about the length of your paper, put length aside. It is most important that you address the question thoroughly – err on the side of thoroughness. That means make sure you answer the question as well and as completely as you can, not just that you add enough filler or rambling that it looks like you did.

Students may work individually or in groups of 2-3. Students should work together in a fair collaboration, with each student contributing equitably to each aspect of the paper. Students working in groups need only hand in one copy of the paper together.

The basic grading breakdown is as follows:

- Mathematics: Analytical, Qualitative, Numerical 50%
- Presentation: Visuals, Layout, Format, Professionalism 25%
- Writing: Exposition, Organization, Clarity, Usage 25%

The details are included in the rubric (below). This rubric page should be submitted as a cover page for your report.

NOTICE: This is a firm reminder (one that should be unnecessary) that academic honesty is required for this (and all) assignments. You should be referring to the ideas in our coursework and course materials to respond to your prompt. You may use third-party resources to *supplement* your work, but you should not look up solutions to these problems and derive your solutions by copying these. If you find interesting quotations or ideas, you must quote them properly and clearly, attributing them to your sources. If you find ideas from a source and use them in your paper, cite your source, even if it's not a quotation.

Besides working within a team, you are not permitted to collaborate with others or copy others' work. When plagiarism occurs, it is usually obvious, and it will land you in trouble. This should go without saying, but I have found that this reminder is sometimes useful.

Project 1 Background Material

We have been learning about derivatives, and in particular, we have studied how to determine the derivative $f'(x)$ given some function $f(x)$. We have discovered many rules and examples that, in combination, will tell us how to find the derivative of many different functions.

However, in scientific and mathematical applications, there are often relationships between certain quantities that are not naturally written as functions. For example, the equation of a circle is $x^2 + y^2 = 1$.

This is called an *implicit* relation. Neither variable is written as a function of the other. However, we might still want to think of y as being sort of like a function of x, even though a circle fails the "horizontal line test" and isn't exactly a function.

Circles do have tangent lines (we might have learned this in geometry class), and that means we can find the slope of those lines and determine the rate of change of this not-quite-a-function. The real question is how to do this using our rules.

We could determine $y(x) = \sqrt{1-x^2}$, treating y as a function $y(x)$, but in general, we cannot rely on being able to solve for y as a function of x. That also only works for the top half of the circle, an important nuance.

But if we pretend y is a function of x, how could we look at $y(x)^2$? Using the chain rule. Recall that the derivative of $f(g(x))$ is $f'(g(x))g'(x)$.

If $f(x) = x^2$ and $g(x) = y(x)$, we get $f'(x) = 2x$ and $f'(g(x))g'(x) = 2(y(x))y'(x)$.

We often find ourselves unhappy with so much notation, so we can write this simply as $2yy'$ instead of $2y(x)y'(x)$. This also helps us remember that y isn't really a function of x.

So if we look at the entire equation $x^2+y^2 = 1$, we can take the derivative of every single part to get $2x + 2yy' = 0$. This allows us to take any point of the function. For example, $(x, y) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is on the circle, and the derivative at that point can be obtained by substituting:

$$2\left(\frac{1}{2}\right) + 2\left(\frac{\sqrt{3}}{2}\right) y' = 0.$$

Solving for y' there gives us $y' = \frac{1}{\sqrt{3}}$ and indeed, in general, we could determine that $y' = \frac{x}{y}$. We cannot easily determine y' as a function of only x – that should seem unlikely, considering we have already conceded that y is not easily expressed as a function of x.

Consider another example: A spherical snowball melts according to its surface area, which means that the rate of change of the surface area will be constant, assuming some constant application of heat or other energy. However, what if we are concerned with the mass (or volume) of the snowball? That will change at a different rate.

In order to understand this, our previous ideas might have forced us to take the volume of a sphere *as a function of surface area*. That seems quite difficult, including fractional exponents and a substantial amount of work to come up with the relationship.

Rather than doing this, we can consider the two simple equations for volume and surface area, as they relate to the radius of the sphere:

$$V = \frac{4}{3}\pi r^3, \qquad A = 4\pi r^2.$$

Interesting observation: As functions, $A(r) = V'(r)$. That's not a coincidence, but it's not what we're worried about right now. In our problem, we haven't even really decided that any of the three A, V, r is a function of the other.

Instead, we can differentiate both of those equations, where we don't think of *any* of these as functions of one another. Indeed, they are all functions *of time*!

$$V' = \frac{4}{3}\pi(3r^2r'), \qquad A' = 4\pi(2rr').$$

Now we might be asked the following: If a snowball melts at a rate of $10cm^2/s$, how is the volume of the snowball changing when its radius is 7?

To do this, we plug in all we know into the equation for A':

$$-10 = 4\pi(2(7)r').$$

Note that A' is negative since it is melting. We can solve this to obtain $r' \approx -0.0568411$. We can use this to determine V' as:

$$V' = \frac{4}{3}\pi\big(3(7)^2(-0.0568411)\big) \approx -35.$$

This topic is discussed at length as Chapter 28. You should feel free to consult this part of the textbook for more.

Project 1 Prompt

You are an auditor for the USDA studying a population of feral goats that escaped from a dairy farm three generations prior. The goat population is known to be restricted by the amount of rubbish (r) in the surrounding area by the implicit equation:

$$10^{-9}p^4 - krp = -r^3 - 70r + 5000,$$

where k is a constant related to the amount of protein in the rubbish.

The population will vary based on a number of factors, but based on this relationship, there is a maximum possible value of p.

Your goal is to assess how the protein content in the food will impact the maximum possible value of p. First, set k to a value between 50 and 100 and graph the curve representing the solutions to this equation.

In order to do this, you can use whatever computational tool you like. If you are having trouble, I suggest the Wolfram Programming Lab: `https://lab.open.wolframcloud.com/app/`

You can use the following code, for example. However, this example uses $k = 125$. Change it to a suitable value.

```
ContourPlot[ 10^(-9) p^4 - 125 r p == -r^3 - 70r + 5000,{r,0,2000},{p,0,50000}]
```

Make use of documentation for the software you've chosen to use. You do not need to be stuck trying to figure out how to use the software – ask if you need help.

Use this image to help you determine what to do next. Your goal is to characterize the value of p where $p' = 0$, indicating the place where p is greatest. Use calculus to find this value of p_{max} when $k = 50$, 60, 70, 80, 90, and 100.

Note that you will want to take derivatives and then set $p' = 0$. You will have much greater difficulty working with the equation until you substitute this value of p'.

Note that you will have to solve several algebraic equations, including eventually an equation involving p that may not be solvable by conventional means. Use a computer to find this value numerical. Using the Wolfram Programming Lab, for example, you would use the command `NSolve`.

Take these six values of (k, p_{max}) and plot them. (In Wolfram language, the command is `ListPlot`.) Is there a relationship between k and p_max?? Do your best to describe what you see. Form a conclusion about the relationship between the protein content in rubbish and the maximum population size for this feral goat herd.

Prepare your report for me, Dr. Myers, your USDA supervisor. Do your best to answer the question thoroughly and completely.

Name(s): ____

Item	Subitem	Points	/	Total
Mathematics	Analytical		/	20
	Qualitative		/	15
	Numerical		/	10
	Other		/	5
	Total		/	50
Presentation	Visuals/Layout		/	12
	Format		/	8
	Professionalism		/	5
	Total		/	25
Writing	Exposition		/	10
	Organization		/	5
	Clarity		/	5
	Usage		/	5
	Total		/	25
	Deductions			
GRAND	TOTAL		/	100

Reminder: Please attach this page to the front of your final hardcopy.