This document contains general instructions for completing this assignment, followed by some background leading up to the project, and then the prompt(s) for the project. The grading rubric is also included.

First, let us remember the purposes of the projects in this course: To foster the use of computer-assisted methods in solving mathematical and scientific problems; To engage in constructive, in-depth problem solving to explore course material; and to practice writing in a technical context.

Your paper should be formatted in a reasonable way. You should include your name, the name of the class, the semester, the assignment, and a title you choose for your paper. ("Math Paper" or "Joe Student" or "Paper 2" is not a title.)

You may type your paper using any software with which you are familiar. Examples of software you could use are MS Word + Equation Editor, LaTeX, MathType, or even Mathematica or Maple (both of which have word processing features). In any event, though, your paper should be typed. You will submit a printed copy of this and any other physical supporting materials.

You will also send a PDF version of this paper, the source document (DOC, DOCX, etc.), and any other electronic materials (e.g. code, image files, etc.) by email. These should be submitted the same day as the hard-copy, the due date. The PDF is important – if you only send the DOCX file, it may not display correctly. Please send both.

Your margins, spacing, font choice, etc. should be standard: Margins should be 1" give or take, font should be a standard font in 12 point, and lines should be 1.5-, or double-spaced. Figures and equations should be included appropriately when needed. If you cannot type all your equations, you can do it the old-fashioned way and leave a blank space, then write them in pen on your print-out.

Your paper should address directly, and in depth, the question in the prompt. You should explore the question/topic in detail. The creativity, thoroughness, and correctness of your response will comprise a major component of your grade. You are required not only to find "the answer," but to discuss the solution, how you found it, and what it says about the problem (which will vary by prompt). Although you are not required to include every possible type of topic we have covered, failing to include a relevant type of method, idea, concept, visual, etc. (especially one explicitly mentioned in the prompt) may result in a deduction if your answer seems incomplete.

For computer-based portions of the work, the instructions below and everything else we've covered so far in class should be enough to get you started. If there are any questions about this, please send me an email.

Your paper should be roughly 2-3 pages in length, or about 700 words. Your paper may be longer, especially if you double-space or use a large number of images. The absolute minimum is 500 words, which should span about 1.5-2 pages (assuming one graphic). Papers below this threshold may be subject to automatic deductions. Your paper should include the appropriate amount (and balance) of mathematics and exposition.

If you are concerned about the length of your paper, put length aside. It is most important that you address the question thoroughly – err on the side of thoroughness. That means make sure you answer the question as well and as completely as you can, not just that you add enough filler or rambling that it looks like you did.

Students may work individually or in groups of 2-3. Students should work together in a fair collaboration, with each student contributing equitably to each aspect of the paper. Students working in groups need only hand in one copy of the paper together.

The basic grading breakdown is as follows:

- Mathematics: Analytical, Qualitative, Numerical 50%
- Presentation: Visuals, Layout, Format, Professionalism 25%
- Writing: Exposition, Organization, Clarity, Usage 25%

The details are included in the rubric (below). This rubric page should be submitted as a cover page for your report.

NOTICE: This is a firm reminder (one that should be unnecessary) that academic honesty is required for this (and all) assignments. You should be referring to the ideas in our coursework and course materials to respond to your prompt. You may use third-party resources to *supplement* your work, but you should not look up solutions to these problems and derive your solutions by copying these. If you find interesting quotations or ideas, you must quote them properly and clearly, attributing them to your sources. If you find ideas from a source and use them in your paper, cite your source, even if it's not a quotation.

Besides working within a team, you are not permitted to collaborate with others or copy others' work. When plagiarism occurs, it is usually obvious, and it will land you in trouble. This should go without saying, but I have found that this reminder is sometimes useful.

Project 2 Background Material

We have been learning about using rectangles and trapezoids to approximate the area under a given function. We have focused on doing this with a function $f(x)$ where we have a formula for f that we can evaluate for any input x. This allowed us to vary the number of rectangles/trapezoids, to improve the accuracy of our estimates.

While these are the simplest possible shapes, they are not the only shapes we can use to obtain area estimates. In particular, if we consider the interval $[a, b]$ split into the partition $x_0, x_1, x_2, x_3, \ldots, x_n$, instead of using n rectangles or n trapezoids, we can take each subinterval $[x_i, x_{i+1}]$ and cap the top of it with something besides a straight line (rectangles and trapezoids both having straight lines on top).

We also discussed that these methods can be used to estimate the area under a function for which we do not have a formula, only a sampling of values. This would be for cases where, for example, we have made experimental observations, rather than having a known function from which to draw values.

In order to do better than a straight line, we will need to use more data points within the interval $[x_i, x_{i+1}]$. But if we have a predetermined set of data points, we cannot subdivide. Instead, we have to use more than two of them per subinterval, effectively reducing n.

For example, if we have five data points, rather than breaking the data up into four groups of two, we could break them up into two groups of three. This sounds worse, since we've gone from $n = 4$ to $n = 2$, but our hope is that what we do with three data points per interval will help somehow.

If we only used two data points, the best we can hope to do is a line, since as we know in geometry, two points define a line (and nothing more than that). With three data points, using our knowledge of algebra, this can define something incrementally better than a line: a parabola. (And yes, if we had even more points, we could do more, but we won't.)

In order to fit a parabola to three points, we consider those points: $(x_i, f(x_i))$, $(x_{i+1}, f(x_{i+1}))$, and $(x_{i+2}, f(x_{i+2}))$. Those three points can be used to determine the values of three coefficients a, b, and c, so that $f(x)$ agrees with a parabola $p(x) = ax^2 + bx + c$ at those three points. We simply solve the following system:

$$\begin{aligned} ax_i^2 + bx_i + c &= f(x_i) \\ ax_{i+1}^2 + bx_{i+1} + c &= f(x_{i+1}) \\ ax_{i+2}^2 + bx_{i+2} + c &= f(x_{i+2}). \end{aligned}$$

Be very careful: In this case, we <u>know</u> the values of x_i, x_{i+1}, and x_{i+2} (and f at those points). The unknown values are the coefficients a, b, c.

For example, consider the following table of five values:

x	0	1	2	3	4
$f(x)$	1	2	7	3	5

We will split the data into two sets of three points:

$$P_1: \qquad \{(0,1),(1,2),(2,7)\}$$
$$P_2: \qquad \{(2,7),(3,3),(4,4)\}$$

The set P_1 leads to the following system of equations:

$$\begin{aligned} a(0)^2 + b(0) + c &= 1 \\ a(1)^2 + b(1) + c &= 2 \\ a(2)^2 + b(2) + c &= 7. \end{aligned}$$

Rewriting that more conventionally, we have:

$$\begin{aligned} c &= 1 \\ a + b + c &= 2 \\ 4a + 2b + c &= 7. \end{aligned}$$

We can solve this (by hand, or using a computer algebra system) to find $a = 2, b = -1, c = 1$. Likewise, our set P_2 gives us a similar system:

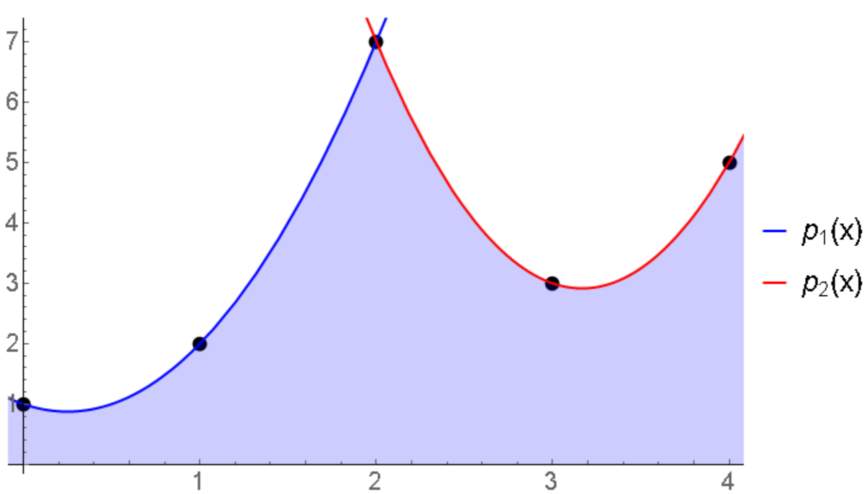
$$\begin{aligned} a(2)^2 + b(2) + c &= 7 \\ a(3)^2 + b(3) + c &= 3 \\ a(4)^2 + b(4) + c &= 5. \end{aligned}$$

which is rewritten:

$$\begin{aligned} 4a + 2b + c &= 7 \\ 9a + 3b + c &= 3 \\ 16a + 4b + c &= 5. \end{aligned}$$

That may be harder to solve by hand, but it's all the same to a computer, and we can obtain the solution fairly easily: $a = 3, b = -19, c = 33$.

We now have two parabolas, $p_1(x) = 2x^2 - x + 1$ and $p_2(x) = 3x^2 - 19x + 33$, which agree with our data in the designated places (p_1 at $x = 0, 1, 2$ and p_2 at $x = 2, 3, 4$). Here is a plot of our data from the table, along with p_1 and p_2.



So instead of approximating the area under some rectangles, we will approximate the area under these parabolas. For this example, we integrate $p_1(x)$ from $x = 0$ to $x = 2$, and then $p_2(x)$ from $x = 2$ to $x = 4$.

$$\begin{aligned}
\int_0^2 (2x^2 - x + 1)dx &= \left[\frac{2}{3}x^3 - \frac{1}{2}x^2 + x\right]_{x=0}^2 \\
&= \left(\frac{2}{3}(2)^3 - \frac{1}{2}(2)^2 + (2)\right) - \left(\frac{2}{3}(0)^3 - \frac{1}{2}(0)^2 + (0)\right) \\
&= \left(\frac{16}{3} - 2 + (2)\right) - (0 - 0 + 0) \\
&= \frac{16}{3}
\end{aligned}$$

$$\begin{aligned}
\int_2^4 (3x^2 - 19 + 33)dx &= \left[x^3 - \frac{19}{2}x^2 + 33x\right]_{x=2}^4 \\
&= \left((4)^3 - \frac{19}{2}(4)^2 + 33(4)\right) - \left((2)^3 - \frac{19}{2}(2)^2 + 33(2)\right) \\
&= (64 - 152 + 132) - (8 - 38 + 66) \\
&= \big(44\big) - \big(36\big) = 8
\end{aligned}$$

The total area is $\frac{16}{3} + 8$, which is $\frac{40}{3}$ or $13.333\ldots$.

That is our approximation of the area under some (unknown) function that has produced the five data points we were given, using parabolas. In this project, you will employ this method of using parabolas to estimate area for a fixed set of data.

It is important to note that there are many commands for recommended computer systems that you can use to help you with this. For example, a slick implementation of this method in the Wolfram language would make use of commands like `Partition` to manipulate your list of data, `InterpolatingPolynomial` to automatically find the equation of each parabola, `Integrate` to perform integration, and `Plot` / `ListPlot` to produce graphics. But you are not required to use these functions – there are many ways to go about this, using commands like `Solve` or doing more of the work by hand.

But if you are curious, all of these may be good places to start, or where to look if you are stuck. As always, refer to the documentation of your chosen software – as well as the internet – for help getting started. If you need help, ask.

Project 2 Prompt

You are stationed at a NASA terrestrial observation platform in the Rocky Mountains. Your station gathers data from space, including radio waves and other phenomena. A new solar phenomenon is creating a certain type of x-ray that your station is observing. You have two pieces of equipment that are capable of observing this phenomenon and measuring the intensity of these waves. The goal is to accurately measure this phenomenon over time and sum up the intensity of these waves over time to determine the cumulative exposure. This means integration.

The first piece of equipment costs over $100,000 per second to run, but generates a nearly true-to-life reading of the phenomenon over time, allowing very precise sampling over time and precise integration. The second piece of equipment costs virtually nothing to run, but only takes two samples per second. Due to budget constraints, we prefer the latter.

Note: The S.I. units of x-ray intensity are joules (J). The integration is over time, measured in seconds, so the result of these computations is joules-seconds (Js).

In order to test and calibrate these instruments, they are being run together to measure the phenomenon. Both machines are now properly calibrated, but you have been asked to consult on how to interpret the data from the second, cheaper machine.

Both systems have analyzed an eight-second sample of this phenomenon. According to the operators assigned to the more expensive system, the total ought to be $16.677Js$. Your goal is to determine how accurate, in this test case, certain types of area-approximation will be using the data. Our hope is to not use the first machine very often.

The data collected by your team are as follows:

Time (s)	Reading (J)	Time (s)	Reading (J)
0.0	3.0000000000	4.5	3.0070391485
0.5	2.9122081864	5.0	0.6962909762
1.0	1.9193049302	5.5	0.2976127359
1.5	1.9303242086	6.0	2.1237437902
2.0	2.2033677913	6.5	3.2159820066
2.5	1.3877110431	7.0	2.4428780954
3.0	0.8094542399	7.5	1.7769832000
3.5	2.1814496707	8.0	2.1362756906
4.0	3.8332122053		

Using this data, estimate the intensity using all the rules we have learned so far: left-endpoint and right-endpoint rectangles, trapezoids, and parabolas. Assess which of these is more accurate. Consider that we may want accuracy up to $0.01Js$ relative to the true value.

Prepare your report for me, Dr. Myers, your NASA supervisor. Do your best to answer the question thoroughly and completely. Please include all your work, justify your calculations, and explain your reasoning.

Do not forget the requirements to submit all original documents, including Word document as DOC, PDF, and hard copy, plus everything else (e.g. code and code output) as PDF. Remember that due to budget cuts, you may only work in groups of 1-3 NASA analysts.

Math 152 **Project 2: Due Nov. 16** **Fall 2017**

Name(s): ______________________________

Item	Subitem	Points	/	Total
Mathematics	Analytical		/	20
	Qualitative		/	15
	Numerical		/	10
	Other		/	5
	Total		/	50
Presentation	Visuals/Layout		/	12
	Format		/	8
	Professionalism		/	5
	Total		/	25
Writing	Exposition		/	10
	Organization		/	5
	Clarity		/	5
	Usage		/	5
	Total		/	25
	Deductions			
GRAND	TOTAL		/	100

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