MTH 152 Quiz 1 – August 31 Fall 2017

Name: Solutions

Please take your time and answer each question clearly and carefully. For this quiz, you will not need a calculator. Do not use one.

A new species of gopher has been discovered. Its population in a particular environment over time is given by the following formula:

$$G(t) = \frac{40t}{5t+1} + 50\left(1 + e^{-0.2t}\right).$$

What is the initial population, $G(0)$?

Hint: Remember that $e^0 = 1$.

$$G(0) = \frac{40 \cdot 0}{5 \cdot 0 + 1} + 50\left(1 + e^{-0.2 \cdot 0}\right)$$

$$= \frac{0}{1} + 50\left(1 + e^0\right)$$

$$= 0 + 50(1+1)$$

$$= 100$$

1. What if we want to know the eventual population size as $t \to \infty$? We will break the problem into two parts. First, consider the fractional part:

2. What is this limit?

$$\lim_{t\to\infty} \frac{40t}{5t+1}$$

Hint: If t is a really big number like a million, what happens?

If t is a million, this becomes

$$\frac{40 \text{ million}}{5 \text{ million and one (approx)}} \approx \frac{40 \text{ mil}}{5 \text{ mil}} = 8$$

Officially:

$$\lim_{t\to\infty} \frac{40t}{5t+1} \cdot \frac{1/t}{1/t} = \lim_{t\to\infty} \frac{40}{5 + 1/t} = \frac{40}{5} = 8$$

$1/t \to 0$ so

TURN OVER

3. What about the other half of $G(t)$?

$$\lim_{t \to \infty} 50(1 + e^{-0.2t})$$

Hint: Don't forget the "50" !

Recall: $\lim_{t \to \infty} e^{-rt} = 0$ (for $r > 0$)

because $e^{-rt} = \frac{1}{e^{rt}}$ and so "$\frac{1}{e^{\infty}}$" is zero.

$$\lim_{t \to \infty} 50(1 + \underbrace{\cancel{e^{-0.2t}}}_{0}) = 50(1+0) = 50$$

4. Add the previous two answers to find $\lim_{t \to \infty} G(t)$.

$50 + 8 = 58$