MTH 152 Quiz 3 – October 24 Fall 2017

Name: Solutions

Please take your time and answer each question clearly and carefully. For this quiz, you will not need a calculator. Do not use one.

1. Find the anti-derivative of each of the following functions. Be sure to carefully apply the appropriate rules for products, exponents, powers, etc. Your answer must include the unknown constant of integration, $+C$.

   (a) If $p(t) = 600e^{6t}$ and $P'(t) = p(t)$, what is $P(t)$?

   $$\int 600e^{6t}\,dt = 600\int e^{6t}\,dt = 600\left(\tfrac{1}{6}e^{6t} + C\right)$$

   $\int kf(x)\,dx = k\int f(x)\,dx$ $\quad$ $\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$

   $$= 600\cdot\tfrac{1}{6}e^{6t} + \boxed{600C} = \boxed{100e^{6t} + \boxed{C_2}}$$

   600 × (any constant) can be any constant

   (b) If $f(q) = 2q + 1$ and $F'(q) = f(q)$, what is $F(q)$?

   $$\int (2q+1)\,dq = 2\int q\,dq + \int 1\,dq = 2\left(\tfrac{1}{2}q^2 + C_1\right) + \tfrac{1}{1}q^1 + C_2$$

   $q^0 = 1$

   $\int (f(x) + g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$ and $\int kf(x)\,dx = k\int f(x)\,dx$ $\quad$ $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$ (twice)

   $$= q^2 + q + \boxed{C_1 + C_2} = \boxed{q^2 + q + \boxed{C}}$$

   (Any number + any other number could be anything.)

   (c) If $g(x) = \sin(x) + x^{-1}$ and $G'(x) = g(x)$, what is $G(x)$?

   $$\int (\sin(x) + x^{-1})\,dx = \int \sin(x)\,dx + \int x^{-1}\,dx = -\cos(x) + C_1 + \ln(|x|) + C_2$$

   (sum rule again) $\quad$ $\int \sin(x)\,dx = -\cos(x) + C$ $\quad$ $\int \frac{1}{x}\,dx = \ln(|x|) + C$

   $$= -\cos(x) + \ln(|x|) + \boxed{C_1 + C_2} = \boxed{-\cos(x) + \ln(|x|) + C}$$

   same as above

TURN OVER

2. (a) Consider the area under the curve $f(q) = 2q + 1$ from $q = 2$ to $q = 4$. Write this using "integral notation" (including the $\int$ sign, etc.).

$$\int_2^4 (2q+1)\, dq$$

q is the variable, so it is "dq".

(b) Find this area using integration. You may refer to your answer to 1(b).

We already know the anti-derivative

$$F(q) = q^2 + q + C.$$

You can ignore the $+C$ (or if you include it, it will cancel out).

Area: $F(4) - F(2) = (4^2 + 4) - (2^2 + 2)$

$= 20 - 6$

$= \boxed{14}$