

Name: Solutions

Please take your time and answer each question clearly and carefully. For this quiz, you will not need a calculator. Do not use one.

1. Solve the following initial-value problem:

$$\frac{dy}{dt} = 1 - y$$

$$y(0) = 1/3$$

$$\frac{1}{1-y} \frac{dy}{dt} = 1$$

$$\int \frac{1}{1-y} dy = \int 1 dt$$

$$u = 1-y$$

$$du = -dy$$

$$\int \frac{1}{u} du = \int 1 dt$$

$$-\ln(|u|) = t + C$$

$$-\ln(|1-y|) = t + C$$

$$\ln(|1-y|) = -t - C$$

$$e^{\ln(|1-y|)} = e^{-t-C}$$

$$1-y = e^{-t} e^{-C}$$

$$y = 1 - e^{-t} e^{-C}$$

$$y = 1 + C_2 e^{-t}$$

solve for  $C_2$

$$y(0) = 1 + C_2 e^0 = 1/3$$

$$1 + C_2 = 1/3$$

$$C_2 = -2/3$$

$$y = 1 - \frac{2}{3} e^{-t}$$

Remember: You don't need +C on both sides, just one.

TURN OVER

2. Solve the following differential equation.

*Note: Remember that the +C will be added on at the step where you do the integration. Depending on how you solve for y, it may not be just a +C tacked on at the end by the time you are done!*

$$y' = t^3 y^2$$

$$\frac{dy}{dt} = t^3 y^2$$

$$\frac{1}{y^2} dy = t^3 dt$$

$$\int \frac{1}{y^2} dy = \int t^3 dt$$

$$-\frac{1}{y} = \frac{1}{4} t^4 + C$$

$$\frac{1}{y} = -\frac{1}{4} t^4 - C$$

$$y = \frac{1}{-\frac{1}{4} t^4 - C}$$

$$y = \frac{-4}{t^4 + 4C}$$

$$y = \frac{-4}{t^4 + C_2}$$

$$(C_2 = 4C)$$