MATH 435 "In-Class" Activity 0 – Due Jan. 16 Spring 2020

Name: Solutions Section: 100

*Please complete the following exercises. Please answer individually (no collaboration).*

1. Simplfy each quantity to the form $a + bi$:

   (a) $(3 + 2i) + (4 - 7i)$

   $$\boxed{7 - 5i}$$

   (b) $(1 + i)^2$

   $$(1+i)(1+i) = 1\cdot 1 + 2\cdot 1\cdot i + i\cdot i = 1 + 2i + i^2$$
   $$= 1 + 2i - 1 = \boxed{2i}$$

   (c) $(1 + i)^{-1}$

   $$\frac{1}{1+i}\cdot\frac{1-i}{1-i} = \frac{1-i}{1^2 - i^2} = \frac{1-i}{1+1} = \boxed{\frac{1}{2} - \frac{1}{2}i}$$

   The conjugate of $1+i$

   (d) $\frac{2i}{3+i}$

   $$\frac{2i}{3+i}\cdot\frac{3-i}{3-i} = \frac{2i(3-i)}{3^2 - i^2} = \frac{6i - 2i^2}{9+1} = \frac{2+6i}{10} = \boxed{\frac{1}{5} + \frac{3}{5}i}$$

   Another conjugate

   (e) $|3 - 4i|$

   $$\sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = \boxed{5}$$

2. Give the roots to the quadratic equation $z^2 - 2z + 5$ as $z = x + iy$.

   It's still just the quadratic formula.

   $$z = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\cdot 1\cdot 5}}{2\cdot 1} = \frac{2 \pm \sqrt{4-20}}{2} = \frac{2\pm\sqrt{-16}}{2} = \frac{2\pm 4i}{2}$$
   $$= \boxed{1 \pm 2i}$$

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3. If $z = x + iy$, describe the relationship between $|z|$ and $z^{-1}$.

   *Hint: Write down each of these in terms of $x$ and $y$.*

$$|z| = \sqrt{x^2+y^2}$$

$$\frac{1}{z} = \frac{1}{x+iy} = \frac{1}{x+iy} \cdot \frac{x-iy}{x-iy} = \frac{x-iy}{x^2+y^2} = \frac{\bar{z}}{|z|^2}$$

Ah ha!

4. (a) Carefully compute the product $(2+3i)(-1+4i)$.

$$2\cdot(-1) + (-1)\cdot 3i + 2\cdot 4i + 3i\cdot 4i$$

$$-2 \quad -3i \quad +8i \quad +12i^2$$

$$-2 \quad +5i - 12 \quad = \boxed{-14+5i}$$

   (b) If you know linear algebra: Compute the product $\begin{pmatrix} 2 & 3 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} -1 & 4 \\ -4 & -1 \end{pmatrix}$.

   *Even if you don't know linear algebra, I bet you can find a tool online that can multiply two matrices.*

The answer will be : $\begin{pmatrix} -14 & 5 \\ -5 & -14 \end{pmatrix}$

The correspondence is:

$$a+bi \longmapsto \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

5. Bonus: Considering the above, how would you relate $|2+3i|$ or $|-1+4i|$ to the respective matrices?

$$|2+3i| = \sqrt{2^2+3^2}$$

$$\det\begin{pmatrix} 2 & 3 \\ -3 & 2 \end{pmatrix} = 2^2+3^2 \quad !$$

neat: $|z|^2 = \det(M_z)$