MATH 435 In-Class Activity 1 – January 16 Spring 2020

Name: Solutions Section: 100

Please complete the following exercises. Please answer individually (no collaboration).

1. Simplify each quantity to the form $a + bi$:

(a) $(3 - 2i) + (1 - 3i)$

$\boxed{4 - 5i}$

(b) $(2 + i)^4$

Start with $(2+i)^2 = 4 + 2\cdot 2\cdot i + i^2 = 4 + 4i - 1 = 3 + 4i$

Now square again: $(2+i)^4 = (3+4i)^2 = 9 + 2\cdot 3\cdot 4i + 16i^2 = \boxed{-7 + 24i}$

(c) $(2 + i)^{-2}$

we already have $(2+i)^2$, so $(2+i)^{-2} = \frac{1}{(2+i)^2} = \frac{1}{3+4i}$

$\frac{1}{3+4i}\cdot\frac{3-4i}{3-4i} = \frac{3-4i}{9+16} = \boxed{\frac{3}{25} - \frac{4}{25}i}$

(d) $\frac{i}{1+i}$

$\frac{i}{1+i}\cdot\frac{1-i}{1-i} = \frac{i - i^2}{1^2 - i^2} = \frac{i+1}{1+1} = \boxed{\frac{1}{2} + \frac{1}{2}i}$

(e) $|1 - 2i|$

$\sqrt{1^2 + (-2)^2} = \sqrt{1+4} = \boxed{\sqrt{5}}$

2. Find the roots of the quadratic equation $z^2 - 2z + 3$.

$z = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\cdot 1\cdot 3}}{2\cdot 1} = \frac{2 \pm \sqrt{4-12}}{2}$

$= \frac{2 \pm \sqrt{-8}}{2} = \frac{2 \pm 2\sqrt{2}\,i}{2} = \boxed{1 \pm \sqrt{2}\,i}$

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3. Use a calculator (or your phone) to determine $e^{\pi i}$. (Please do not use this device for any other problem.) Does the answer surprise you?

$e^{\pi i} = -1$

Your level of surprise will vary.

4. (a) Carefully compute the product $(3+4i)(1-i)$.

$= 3\cdot 1 - 3\cdot i + 4i\cdot 1 - 4i\cdot i$

$= 3 - 3i + 4i + 4$

$= \boxed{7+i}$

(b) Find $|3+4i|$ and $|1-i|$. Then find the absolute value (now also known to us as "modulus") of your answer to the previous qustion.

$|3+4i| = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

$|1+i| = \sqrt{1^2+1^2} = \sqrt{2}$

$|7+i| = \sqrt{7^2+1^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$

(c) How do your previous answers compare?

We see the rule $|z_1 z_2| = |z_1|\cdot|z_2|$ in action.