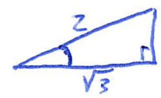
MATH 435 **In-Class Activity 2 – January 23** **Spring 2020**

Name: Solutions Section: 100

Please complete the following exercises. Please answer individually (no collaboration).

1. Write each number in polar form, $z = re^{\theta i}$:

Reference triangle:



"30-60-90" ↳ $\pi/6$

(a) $-\sqrt{3}+i$

Take care!

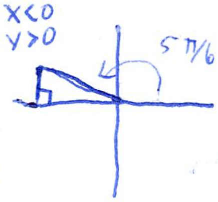
Be sure your arctans are in the right quadrant!

$$r = |z| = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$$

$$\theta = \arctan\left(-1/\sqrt{3}\right) = 5\pi/6 \quad (\text{not } -\pi/6)$$

$$\boxed{z = 2e^{5\pi/6\, i}}$$

x<0 y>0



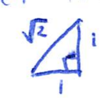
(b) $4-4i$

$$r = |z| = \sqrt{4^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$$

$$\theta = \arctan(-4/4) = -\pi/4 \quad (\text{not } 3\pi/4)$$

$$\boxed{z = 4\sqrt{2}\, e^{-\pi/4\, i}}$$

Ref triangle:



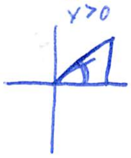
"45-45-90" ↳ $\pi/4$

x>0 y<0

2. Write each number in standard form, $z = x + iy$:

(a) $7e^{\pi i/4}$

x>0 y>0

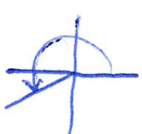


$$x = r\cos\theta = 7\cos(\pi/4) = 7\cdot\frac{1}{\sqrt{2}}$$

$$y = r\sin\theta = 7\sin(\pi/7) = 7\cdot\frac{1}{\sqrt{2}}$$

$$\boxed{z = \frac{7}{\sqrt{2}} + \frac{7}{\sqrt{2}}\, i}$$

(b) $2e^{7\pi i/6}$



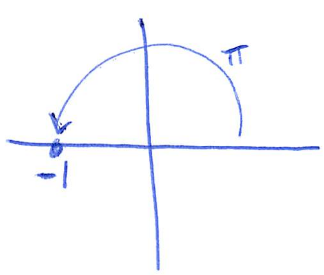
x<0 y<0

$$x = r\cos\theta = 2\cos(7\pi/6) = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$$

$$y = r\sin\theta = 2\sin(7\pi/6) = 2\left(-\frac{1}{2}\right) = -1$$

$$\boxed{z = -\sqrt{3} - i}$$

3. Remind yourself using a calculator, or by drawing a diagram, or by some other means, of what $e^{\pi i}$ is. Does this now make more sense than it did last time we saw it?

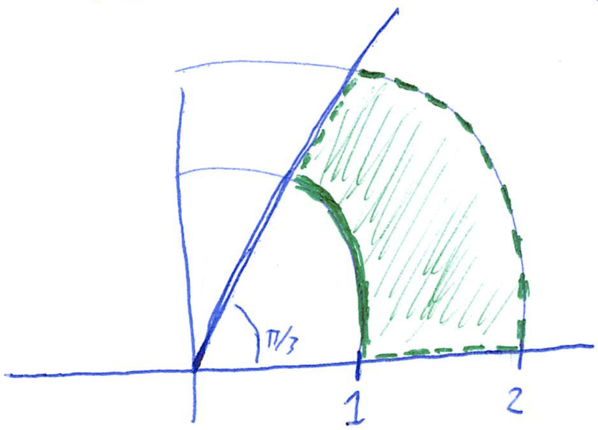


I hope the geometry and connection to polar coordinates is now clearer.

OVER

4. Describe the region in the complex plane with z such that $1 \leq |z| < 2$ and $0 < \mathrm{Arg}(z) < \pi/3$. Sketch a picture, but also describe in a few words as necessary.

$|z|$ restricts the radius, $\mathrm{Arg}(z)$ the angle.



← It only contains part of its boundary.

5. Find the roots indicated

(a) $(1+i)^{1/2}$

Step 1: convert to polar form $z = \sqrt{2}\, e^{\pi/4 i}$

Step 2: write out enough roots (2) to get all: $z^{1/2} = \left(\sqrt{2}\, e^{\pi/4 i}\right)^{1/2} = \left(\sqrt{2}\, e^{9\pi/4 i}\right)^{1/2}$

Step 3: Add the root exponents ↗

Step 4: Simplify the exponents: $z^{1/2} = (\sqrt{2})^{1/2} e^{\pi/8 i}$ or $(\sqrt{2})^{1/2} e^{9\pi/8 i}$

$\boxed{\sqrt[4]{2}\, e^{\pi/8 i} \text{ or } \sqrt[4]{2}\, e^{9\pi/8 i}}$

Step 5 (optional/contextual): Put back into rectangular form

$$z = \sqrt[4]{2}\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right) + \sqrt[4]{2}\left(\frac{\sqrt{2-\sqrt{2}}}{2}\right) i$$

(or the negative of this)

(b) $(-8 - 8\sqrt{3}i)^{1/4}$

Try to get this one on your own!

If you can't, come talk to me!