

Name: \_\_\_\_\_

Section: 100

*Please complete the following exercises. You may collaborate with your classmates, consult your notes or text, and/or ask for help. Note that participation in this activity is not optional.*

1. State where each complex function is not continuous, or say that it is continuous everywhere.

(a)  $h(z) = \frac{z^3 - i}{z - i}$

(b)  $f(z) = z^2$

(c)  $g(z) = \frac{1}{z-1} + \frac{1}{\bar{z}-i}$

2. Find the derivative of each complex function. (You should use the rules from §20.)

(a)  $q(z) = (2 - z)^6$

(b)  $u(z) = (1 - z)^3(z + 2)$

(c)  $v(z) = \frac{(z-1)^2}{z^2+2}$

3. Consider the function  $p(z) = 2z^2 - 3z + 1$ , and let  $\gamma$  represent the unit circle  $|z| = 1$ .

(a) Explain why  $p(z)$  achieves a maximum value on  $\gamma$ . Cite a specific theorem from the text.

(b) Write  $z$  in polar form when  $|z| = 1$ .

(c) Write  $|p(z)|$ , restricted to  $\gamma$ , as a function of  $\theta$ . What type/range of number is allowed as the input of  $\theta$ ? What kind of output do you get from  $|p(z)|$ ?

(d) Use the triangle inequality to obtain a bound for  $|p(z)|$  on  $\gamma$ .

(e) Bonus: Can you guess, approximate, or find the exact maximum value of  $|p(z)|$  on  $\gamma$ ? Or can you guess, approximate, or find the value of  $\theta$  that achieves this maximum?