MATH 435 **In-Class Activity 5 – February 10** **Spring 2020**

Name:_____ Section: 100

Please complete the following exercises. You may collaborate with your classmates, consult your notes or text, and/or ask for help. Note that participation in this activity is not optional.

1. Confirm that each function satisfies the Cauchy-Riemann equations using the polar version.

   (a) $f(z) = \dfrac{1}{z^3}$

   (b) $g(z) = z^n$ (you may assume n is a positive integer)

2. Verify that the following function is entire: $f(x+iy) = \big(x+x^3-3xy^2\big)+\big(y-y^3+3x^2y\big)i$.
   Be sure to do the required computation(s), but also to note that all conditions are met.

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3. Consider the function $h(z) = \frac{1}{z(z^2+1)}$.

   (a) Where is $h(z)$ analytic? Give the largest possible domain.

   (b) Where are the singular points of $h(z)$?

   (c) What would we normally call the point $x = 0$ for the real function $h(x)$?

4. We discussed $f(z) = |z|^2$ as a function that has a derivative at only one point. (If you forget, the point is $z_0 = 0$.) Is it analytic at this point? Why or why not?