MATH 435 In-Class Activity 6 – February 27 Spring 2020

Name: Solutions Section: 100

Please complete the following exercises. You may collaborate with your classmates, consult your notes or text, and/or ask for help. Note that participation in this activity is not optional.

Recall: $\log(z) = \ln|z| + i\arg(z)$

1. Simplify each quantity (you may use polar form for convenience).

   (a) i^i

   $e^{i \log(i)} = e^{i \cdot (\pi i/2)} = \boxed{e^{-\pi/2}}$

   $i = 1 \cdot e^{\pi i/2}$

   $\log(i) = \ln(1) + \pi i/2 = \pi i/2$

   (b) $i^{\pi i}$

   You can repeat the above, just be careful with the extra π.

   $i^{\pi i} = e^{\pi i \log(i)} = e^{\pi i (\pi i/2)} = \boxed{e^{-\pi^2/2}}$

   (c) $(1+i)^i$

   $e^{i \log(1+i)} = e^{i(\ln\sqrt{2} + i\pi/4)} = \boxed{e^{-\pi/4 + i\ln\sqrt{2}}}$

   (d) $(-1)^{1/\pi}$

   $e^{\frac{1}{\pi}\log(-1)} = e^{\frac{1}{\pi}(\pi i)} = \boxed{e^{i}}$

   (e) $(-1)^{2/\pi}$

   $e^{\frac{2}{\pi}\log(-1)} = e^{\frac{2}{\pi}\cdot \pi i} = \boxed{e^{2i}}$

Note: In class, we decided to do just principle values together here. Notice how they might all change for different values of logs.

OVER

2. Consider the function $f(z) = z^\alpha$ for some unknown number α. (Assume the "worst case," meaning α is not an integer or anything special.)

(a) Write $f(z)$ using exponential/logarithmic functions.

$$f(z) = e^{\alpha \log(z)}$$

(b) Where is $f(z)$ analytic?

It depends. The multi-valued version with "log" is analytic everywhere, but it's a multi-valued function.

The single-valued principle branch (or any other branch) is analytic except along the branch cut.

(c) What happens if you restrict x to a real number and consider $f(x)$?

$$f(x+0y) = e^{\alpha \log(x+0y)} = e^{\alpha \ln(x)} = x^\alpha$$

(ignoring multiple values)

(d) Explain the difference between the multivalued and principle valued version of $f(z)$.

(see above - your answers for b and c would have to be reconsidered based on which you gave.)